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Proper-time foundations for classical electrodynamics

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Abstract In this paper, we provide an introduction to the Howard University research program on the canonical proper-time formulation of classical electrodynamics Our approach leads to a new set of Maxwell's equations which fixes the clock of the field source for all inertial observers but is mathematically equivalent to the conventional theory However, the speed of light is no longer an invariant for all observers, but depends on the motion of the source (Thus, a fundamental conclusion is that mathematical equivalence is not always related to physical equivalence) This approach allows us to account for radiation reaction without the use of mass renormalization or advanced potentials This means that all the problems associated with the Lorentz-Dirac equation do not occur In addition, no assumptions are required about the structure of the source The theory also provides a new invariance group which, in general, is a nonlinear and nonlocal representation of the Lorentz group This approach provides a natural (and unique) definition of simultaneity for all observers, furthermore, there is no physical advantage in using time as a fourth coordinate (although it is still a fourth dimension)

Keywords Special relativity, proper time, radiation reaction

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1. Introduction

In this paper, we provide an outline of the research program at Howard University on the development of a completely physically motivated representation of classical electrodynamics. Our philosophy is based on the following assumptions (or beliefs) .

(1) Mathematics provides a set of tools for constructing faithful representations of physical reality but does not dictate the final outcome.

(2) Given the infinite number of possible mathematical tools and structures possible, there is no a priori reason that we cannot construct representation(s) of physical

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reality that corresponds to the way the world appears to us in our consciousness.

(3) The current (intellectual) state of affairs in physics is not due to past technical mistakes but those of a conceptual nature.

The specific goals of our program are :

(1) To use and/or develop mathematics that is clearly motivated by physics or clearly stated philosophical (physical) principles.

(2) To carefully study the historical, conceptual and philosophical background to both classical electrodynamics and relativistic quantum theory in order to identify all open problems and/or unanswered questions as clearly discussed by the founding fathers.

(3) To solve the problems or answer the questions of goal two.

(We will briefly discuss our work on relevant mathematical issues and relativistic quantum theory at the end of the paper.) Here, our focus is on the classical theory.

2. Background (History)

Einstein begins his 1905 [1], paper with the statement :

It is known that Maxwell's electrodynamics as usually understood at the present time – when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena.

After quoting a few examples and the unsuccessful attempts of experimenters to discover the light medium (ether), he concludes that mechanics as well as electrodynamics possess no properties corresponding to absolute rest. Thus, the laws of electrodynamics and optics will be valid for all frames in which the equations of mechanics hold. He then suggests that we raise his conjecture to the status of a postulate called the "principle of relativity" He then adds one other postulate to provide what are now known as the basic postulates of the special theory of relativity :

(1) The physical laws of nature and the results of all experiments are independent of the inertial frame of the observer.

(2) The speed of light (relative to all inertial observers) is constant.

Today, after such a long time, many assume that there is ample experimental evidence to support this second assumption. However, the very nature of a postulate means that *it is a basic assumption of the theory,* presented without proof. In fact, if proof were available, the postulate would not be needed. Indeed, as Einstein rightly points out in the first footnote to his second paper [2] : "The principle of the constancy of the velocity of light is of course contained in Maxwell's equations". What he meant by this was that the second postulate follows from the fact that the constant c in Maxwell's equations (as currently formulated) is an invariant for all (inertial) observers. Since that time, many experiments have been done to verify this assumption (Experiments have verified that the speed of light from a source at rest in an inertial frame is constant with value c) However, in 1965, Fox [3] wrote a very important paper which reviewed the evidence for constant c and against the emission theory of Ritz [4] His conclusion was that all previous experiments were flawed for a number of reasons In many cases, analysis of the experimental data failed to take into account the (now well-known) extinction theorem of Ewald and Oseen (see Jackson [5]) The only data found that firmly supported the second postulate came from experiments on the lifetime of fast mesons and the velocity of γ rays and light from moving sources In his conclusion Fox states that

> Unless something has been overlooked, these seem to be the only pieces of experimental evidence we have This is surprising in light of the long history and importance of the problem

(We will return to this later and show that the experiments on the lifetime of fast mesons and the velocity of γ rays and light from moving sources must be re-evaluated)

As noted by Bridgman [6], the special theory allows us to by-pass but not answer the fundamental question of "the nature of the physical mechanism by which objects are lighted" From an operational point of view we must ask if it is physically possible to consider light as a "thing" that travels ? Bridgman [6] observed that

> We can give no operational meaning to the idea that light exists at each point between source and sink The idea of light as a thing travelling is pure invention based on sense perceptions and the mechanical world view

Bridgman further points out that the special theory of relativity spreads time over space by assuming light is a thing travelling Hence, if we assume that light is the transfer of energy, conservation of energy requires that we integrate the local energy density over all space at a definite time instant, which puts us in a logical circle, as this implies the non-local nature of light

As noted in Miller [7], Einstein chose to consider light as a thing travelling for convenience This allowed him to use the standard notion of velocity for measurement purposes However, in the special theory, light is not a material particle nor is it a wave, since if it's a particle its velocity cannot be independent of the source motion and, if it's a wave, it must travel in a medium (the ether), which is known to not have any effect on light !

It should not go unnoticed that, in a paper published almost at the same time (a few months later), Einstein [8] used the concept of light as a "localized energy packet" to explain the photoelectric effect In fact, Planck [9] wrote

> According to the latest statements by Einstein it would be necessary to assume that free radiation in vacuum, and hence light waves themselves, has an atomistic constitution, and thus to abandon Maxwell's equations

We should not be amazed at Planck's statement since, at the time, the question of the need for Maxwell's equations at all was still an open subject

In 1867, Ludvig Lorentz [10] introduced the retarded vector and scalar potentials It was shown that these led to the same results obtained by Maxwell *via* the introduction of the displacement current into Ampére's law Indeed, it has been known since then that all the results of the Maxwell theory can be obtained directly from the potentials, without ever introducing fields (It has recently been shown by Hamdan, Hariri and Lopez-Bonilla [39] that one can derive Maxwell's equations directly from the Lorentz force)

There were many who took L Lorentz's position, but the major protagonist in this debate was Walther Ritz [4] Ritz, like Einstein, accepted H A Lorentz's theory of the electron but rejected the ether He further noted that, from a strictly logical point of view,Maxwell's electric and magnetic fields, which appear to play such an important role can be entirely eliminated from the theory He argued that, in reality, Maxwell's theory deals only with certain relations between space and time In his view, we could simply return to the elementary actions (retarded potentials) He further pointed out that the field equations had an infinite number of solutions that are incompatible with experiment and in order to eliminate these extraneous solutions, it is necessary to adopt the retarded potentials anyway This introduces an additional assumption which is not needed if we start with the retarded potentials in the first place

Einstein did not completely accept, but was swayed by Ritz's position Indeed, in his 1909 paper [11], Einstein stated

According to the usual theory, an oscillating ion generates a divergent spherical wave The reverse process does not exist as a elementary process The convergent wave is indeed mathematically possible, but for its approximate realisation an enormous number of elementary emitting elementary systems would be required Hence the elementary process of light-emission has not as such the character of reversibility Herein, I believe our wave theory is incorrect It seems in relation to this point Newton's emission theory contains more truth then the wave theory, for the energy communicated to a light-particle in emission is not spread over infinite space but remains available for an elementary process of absorption

Here, Einstein is agreeing with Ritz's position that retarded potentials express the elementary process of emission, whereas Maxwell's equations do not We get a further clue to Einstein's thinking on this subject from his *Autobiographical notes* of 1949 [12] (see Brown [13])

> Reflections of this type made it clear to me as long ago as 1900, *i e*, shortly after Planck's trailblazing work, that neither mechanics nor electrodynamics could (except in limiting cases) claim exact validity

Brown points out that, because he was not sure that Maxwell's theory would survive the existence of photons, Einstein derived the Lorentz transformations from kinematical arguments, as opposed to the symmetry properties of Maxwell's equations. He believed that the Lorentz transformations were fundamental and would survive any failures in the Maxwell theory.

In the past, there always was a certain tension between field theory and action-at-a-distance. The most famous recent work on the subject is the Wheeler-Feynman formulation of classical electrodynamics [14], in which they eliminate the field completely (in favour of an action-at-a-distance approach) in order to solve the self-energy divergence problem associated with the then accepted Dirac theory [15]. However, among other things, the need for both advanced and retarded interactions, the inability to quantize and the intrinsic usefulness of the self-energy divergence for the success of quantum electrodynamics became important reasons for its lack of favor as a replacement for the Dirac approach.

2 1. *Purpose :* It is important to remember that the special theory came about because of differences that occurred when Maxwell's equations were applied to bodies in motion, compared to those at rest. In the first section we provide an introduction to the canonical proper-time formulation of classical electrodynamics, where the local clock of the moving system replaces the clock of the observer. This approach is mathematically equivalent but is not physically equivalent. Physically, this change is equivalent to a new definition of velocity for relativistic systems. In the second section we look at few areas where additional assymptions are required in order to rescue the conventional theory and explain new findings. In the third section, we provide a brief review of our research on the foundations of relativistic quantum theory. We discuss open problems associated with the Dirac equation, the square-root equation and the Feynman path integral.

3. Canonical proper-time classical theory

3 1 Observers and observed systems :

In actual experimental setups there is an observer and a system to be observed. (The observer has his/her own inertial frame of reference, including clocks and measuring equipment.) There are few (if any) experiments of interest conducted on systems with constant velocity. In general, some interaction is required, so that the system responds to forces. After sufficient data is obtained and analyzed (based on current theoretical guildelines) a report of the findings is prepared. There are the essentials of the process. The first postulate of the special theory of relativity imposes a natural constraint on the extent that we may believe in the results of the experiment; namely, that any other observer, using similar equipment in any other inertial frame of reference must be able to obtain results that differ, at most, by a Lorentz transformation.

It was natural for Einstein to use the clock of the observer to measure time. The recognition that this constraint on theory is a convention is a major thesis of our research program. In the following section, we show that an equally valid clock to use is the clock of the observed system, which is generally known as the proper-time. (In this terminology, the conventional clock used is the proper-time of the observer.)

3.2. Canonical proper-time particle theory :

The key concept of the (classical) canonical proper-time theory for particles may be seen by examining the time evolution of a dynamical parameter W $(x,\ p)$ *via* the standard formulation of classical mechanics, described in terms of the Poisson brackets :

$$\frac{dW}{dt} = \{H, W\}. \tag{1}$$

We can also represent the dynamics using the proper (or local) time of the system. To do this, recall that the proper-time has the representation $d\tau = (1/\gamma)dt = \left(mc^2/H\right)dt$, so that :

$$\frac{dW}{d\tau} = \frac{dt}{d\tau}\frac{dW}{dt} = \frac{H}{mc^2}\{H, W\}. \tag{2}$$

Assuming a well-defined (invariant) rest energy (mc^2) for the system, we determine the canonical proper-time Hamiltonian K such that :

$$\{K, W\} = \frac{H}{mc^2}(H, W), \quad K|_{p=0} = H|_{p=0} = mc^2.$$

Using

$$\{K, W\} = \left[\frac{H}{mc^2}\frac{\partial H}{\partial p}\right]\frac{\partial W}{\partial x} - \left[\frac{H}{mc^2}\frac{\partial H}{\partial x}\right]\frac{\partial W}{\partial p}$$

$$= \frac{\partial}{\partial p}\left[\frac{H^2}{2mc^2} + a\right]\frac{\partial W}{\partial x} - \frac{\partial}{\partial x}\left[\frac{H^2}{2mc^2} + a'\right]\frac{\partial W}{\partial p}$$

we get that $a = a' = \frac{1}{2}mc^2$, so that (assuming no explicit time dependence)

$$K = \frac{H^2}{2mc^2} + \frac{mc^2}{2}, \quad \text{and} \quad \frac{\partial W}{\partial \tau} = (K, W). \tag{3}$$

3.3. Many-particle case :

Suppose we have a closed system of n interacting particles, with individual Hamiltonians H_i and total Hamiltonian H. We assume that $H = \sum_{i=1}^{n} H_i$ and, if we define the effective mass M and total momentum $\boldsymbol{P}$ by $\boldsymbol{P} = \sum_{i=1}^{n} \boldsymbol{p}_i$ and $Mc^2 = \sqrt{H^2 - c^2\boldsymbol{P}^2}$, we get

$H=\sqrt{c^2\boldsymbol{P}^2+M^2c^4}$. Now use our definition of proper-time to obtain $d\tau=\left(Mc^2/H\right)dt$ and $d\tau_i=\left(M_ic^2/H_i\right)dt$. It is easy to show that

$$K=\frac{H^2}{2Mc^2}+\frac{Mc^2}{2}\qquad 2M\quad +Mc^2$$

and, for any observable W, we have

$$\frac{dW}{d\tau}=\{K,W\}=\sum\nolimits_{i=1}^{n}\frac{d\tau_i}{d\tau}\{K_i,W\}. \tag{4}$$

Note that the position and momentum variables in K, K_i, are the ones measured in the observer's inertial frame and not in the proper frame of the system (which would measure zero momentum). This shows that our transformation is not a Lorentz transformation of the dynamical parameters of the system. To show explicitly that the transformation is a canonical change of variables (time), set $S=\left[Mc^2-K\right]\tau$. An easy calculation, using the fact that both Mc^2 and K are conserved quantities, shows that $dS=\left[Mc^2-K\right]d\tau$. It follows that :

$$\boldsymbol{P}\cdot d\boldsymbol{X}-Hdt\equiv\boldsymbol{P}\cdot d\boldsymbol{X}-Kd\tau+dS\,,$$

$$\sum_{i=1}^{n}p_i\cdot dx_i-\sum^{n}H_idt\equiv\sum^{n}p_i\cdot dx_i-Kd\tau+dS\,.$$

3.4. Time reversal noninvariance :

Since (in the single particle case) $d\tau=\left(mc^2/H\right)dt$ and, as $K=\left[H^2/2mc^2+mc^2/2\right]$ and m are always positive, we see that if $t\to-t$ (time reversal) or $H\to-H$, then $K\to K$ is invariant, while $\tau\to-\tau$. *Thus our theory is noninvariant under time reversal at the classical level and, since τ is monotonically increasing, we acquire an arrow for (proper) time.* It is thus natural to interpret anti-matter as matter with its proper-time reversed. A more complete (and elegant) discussion requires the introduction of Santilli's isodual numbers [16], in which the unit 1 is replaced by -1 and $ab\to a*b=-ab$ so that $(-1)*(-1)=-1$ (see Gill *et al* [17]). Thus, by introducing a symmetric theory of numbers, we can construct a completely symmetric theory of matter which avoids all of the natural objections to hole theory, while maintaining consistency with our physical sense of a monotonically increasing time variable. Both Feynman [18] and Stueckelberg [19] introduced the idea of representing anti-matter as matter with its time reversed. Our final conclusion is the same as theirs. However, the two approaches are distinct. In our approach, we replace t by τ and acquire K as its cannonical Hamiltonian, so all physical interpretations only require information about τ .

The quantum theory now follows by replacing the Poisson bracket in eq. (4) by the Heisenberg bracket, which leads to a Schrödinger-like equation :

$$i\hbar\frac{\partial\psi}{\partial\tau} = K\psi\,, \text{ and } \quad i\hbar\frac{\partial\psi}{\partial\tau_i} = K_i\psi\,,$$

for the same (universal) wave function ψ. Since K, K_i are both positive definite, the problems which caused confusion during the early attempts to merge quantum mechanics and the special theory of relativity do not arise. The question of particle number is easily included (even in the classical case) by observing that, for any closed system of interacting particles, we can replace the definite particle number n by a variable (random) particle number $N(t)$, the number of particles up to time t (as seen by the observer), with the constraint that the total global energy, momentum, angular momentum and spin remain constant and, as in QED, for large negative t, $N(t) \to n_i$, (the initial particle number), and for large positive t, $N(t) \to n_f$, (the final particle number).

3.5. Maxwell theory :

In order to formulate the proper-time version of the Maxwell theory, it is convenient to start with the standard definition of proper-time :

$$d\tau^2 \quad dt^2 - \frac{1}{c^2}dx^2 = dt^2 \left| 1-\frac{w^2}{c^2} \right. \qquad w = \frac{dx}{dt}$$

Motivated by geometry, Minkowski introduced the concept of proper – time. Recently, it has been suggested by Damour [20] that Minkowski was not aware that $d\tau$ is not an exact one-form and hence cannot be used for a metric. Thus, he did not completely understand its physical meaning, since a major conclusion of Einstein was that a moving system measures time differently compared to one at rest (relative to an observer). (For very interesting additional discussion on this and other related points, see Walters [21] and included references.) It is clear that Minkowski became aware of this fact eventually, if Sommerfeld is to be believed (see his notes in [22] after the translation of Minkowski's paper (p. 94)).

Nevertheless, some of the mathematically inclined have dismissed this (physical) fact by attaching a "co-moving observer" on the tangent curve (bundle) of the moving particle in order to induce an instantaneous exact one-form for the four-geometry at each time slice. (This is mathematically correct but physical nonsense.) However, there is an important physical reason why $d\tau$ is not an exact (mathematical) one-form. Physically, a particle can traverse many different paths (in space) during any given τ interval. This reflects the fact that the distance travelled in a given time interval depends on the forces acting on the particle. This suggests that the actual clock of the source carries additional physical information, and there is no a *priori* (physical) reason to believe that this information is properly encoded in the clock of a mathematical co-moving observer. In order to see that there is (indeed) additional physical information, rewrite the above equation as :

$$dt^2 = d\tau^2 + \frac{1}{c^2}dx^2 = d\tau^2\left|1+\frac{\boldsymbol{u}^2}{c^2}\right. \qquad \boldsymbol{u} = \frac{dx}{d\tau}.$$

This suggests a certain duality in the relationship between t, τ and $\boldsymbol{w}$, $\boldsymbol{u}$. To see that this is indeed the case, recall that $\boldsymbol{u} = \boldsymbol{w}\big/\sqrt{1-\left(w^2/c^2\right)}$. Solving for $\boldsymbol{w}$, we get that $\boldsymbol{w} = \boldsymbol{u}\big/\sqrt{1+\left(\boldsymbol{u}^2/c^2\right)}$. If we set $b = \sqrt{c^2+\boldsymbol{u}^2}$, this relationship can be written as

$$\frac{\boldsymbol{w}}{c} = \frac{\boldsymbol{u}}{b} \tag{5}$$

For reasons to be discussed later, we call b the collaborative speed of light. We also note that

$$\frac{1}{c}\frac{\partial}{\partial t} = \frac{1}{c}\frac{\partial\tau}{\partial t}\frac{\partial}{\partial\tau} = \frac{1}{c}\frac{1}{\sqrt{1+\left(\boldsymbol{u}^2/c^2\right)}}\frac{\partial}{\partial\tau} = \frac{1}{b}\frac{\partial}{\partial\tau} \tag{6}$$

In c.g.s. units Maxwell's equations have the form

$$\nabla\cdot\boldsymbol{B} = 0, \qquad \nabla\cdot\boldsymbol{E} = 4\pi\rho, \tag{7}$$

$$\nabla\times\boldsymbol{E} = -\frac{1}{c}\frac{\partial\boldsymbol{B}}{\partial t}, \quad \nabla\times\boldsymbol{B} = \frac{1}{c}\left[\frac{\partial\boldsymbol{E}}{\partial t} + 4\pi\,\rho w\right.$$

Using eqs. (5) and (6) in (7), we have (*the identical mathematical representation for Maxwell's equations*) :

$$\nabla\cdot\boldsymbol{B} = 0, \qquad \nabla\cdot\boldsymbol{E} = 4\pi\rho$$

$$\nabla\times\boldsymbol{E} = -\frac{1}{b}\frac{\partial\boldsymbol{B}}{\partial\tau}, \quad \nabla\times\boldsymbol{B} = \frac{1}{b}\left.\frac{\partial\boldsymbol{E}}{\partial\tau} + 4\pi\,\rho\boldsymbol{u}\right]. \tag{8}$$

Thus, we see that Maxwell's equations are equally valid when the local time of the particle is used to describe the fields. This leads to the following important conclusions :

(1) There are two distinct clocks to use in the representation of Maxwell's equations. (Thus, the choice of clocks is a convention in the true sense of Poincaré).

(2) Since the two representations are mathematically identical, we conclude that mathematical equivalence is not physical equivalence. (This will be absolutely clear after we derive the corresponding wave equation below.)

(3) When the local clock of the system is used, the constant speed of light c is replaced by the collaborative speed of light b, which depends on the motion of the system (*e.g.*, $b = \sqrt{c^2 + \boldsymbol{u}^2}$).

(4) There is another group (closely related to the Lorentz group) which fixes the proper-time of the particle for all observers.

Before constructing the group, we first derive the corresponding wave equations in the proper-time variable. Taking the curl of the last two equations in (8), and using standard vector identities, we get :

$$\frac{1}{b^2}\frac{\partial^2 \boldsymbol{B}}{\partial \tau^2} - \frac{\boldsymbol{u}.\boldsymbol{a}}{b^4}\frac{\partial \boldsymbol{B}}{\partial \tau} - \nabla^2 \cdot \boldsymbol{B} = 1\left[4\pi\nabla \times (\rho \boldsymbol{u})\right.$$

$$\frac{1}{b^2}\frac{\partial^2 \boldsymbol{E}}{\partial \tau^2} - \frac{\boldsymbol{u}\cdot\boldsymbol{a}}{b^4}\frac{\partial \boldsymbol{E}}{\partial \tau} - \nabla^2 \cdot \boldsymbol{E} = -\nabla(4\pi\rho) - \frac{1}{b}\frac{\partial}{\partial \tau}\left[4\pi\nabla \times (\rho \boldsymbol{u})\right. \tag{9}$$

where $a = d\boldsymbol{u}/d\tau$ is the collaborative acceleration caused by external forces. Thus, we see that a new term arises when the proper-time of the system is used to describe the fields. This makes it clear that the local clock encodes information about the particle's interaction that is unavailable when the clock of the observer is used to describe the fields, and shows clearly that physical equivalence is not the same as mathematical equivalence. The new term in eq. (9) is dissipative, acts to oppose the acceleration, is zero when $\boldsymbol{a}$ = 0, and arises instantaneously with the action of forces on the particle This is exactly what one expects of the back reaction caused by the inertial resistance of the particle to accelerated motion and, according to Wheeler and Feynman [14], is precisely what is meant by radiation reaction. Thus, the collaborative use of the observer's coordinate system and the local clock of the observed system provides intrinsic information about the field dynamics not available in the conventional formulation of Maxwell's theory. If we make a scale transformation (at fixed position) with $\boldsymbol{E} \to (b/c)^{1/2}\boldsymbol{E}$ and $\boldsymbol{B} \to (b/c)^{1/2}\boldsymbol{B}$ equations (9) transform to

$$\frac{1}{b^2}\frac{\partial^2 \boldsymbol{B}}{\partial \tau^2} - \nabla^2.\boldsymbol{B} + \left(\frac{\ddot{b}}{2b^3} - \frac{3\dot{b}^2}{4b^4}\right)\boldsymbol{B} = c^{\frac{1}{2}}\frac{4\pi}{b^{\frac{3}{2}}}\nabla \times (e'u),$$

$$\frac{1}{b^2}\frac{\partial^2 \boldsymbol{E}}{\partial \tau^2} - \nabla^2.\boldsymbol{E} + \left|\frac{\ddot{b}}{2b^3} - \frac{3\dot{b}^2}{4b^4}\right|\boldsymbol{E} = -\frac{c^{1/2}}{b^{1/2}}\nabla(4\pi\rho) - \frac{c^{1/2}}{b^{3/2}}\frac{\partial}{\partial \tau}\left[\frac{4\pi \times (\rho \boldsymbol{u})}{b}\right]. \tag{10}$$

This is the Klein-Gordon equation with an effective mass μ given by

$$\mu = \left\{\frac{\hbar^2}{c^2}\left[\frac{\ddot{b}}{2b^3} - \frac{3\dot{b}^2}{4b^4}\right]\right\}^{1/2} = \left\{\frac{\hbar^2}{c^2}\frac{\boldsymbol{u}\cdot\ddot{\boldsymbol{u}} + \dot{\boldsymbol{u}}^2}{2b^4} - \frac{5(\boldsymbol{u}.\dot{\boldsymbol{u}})^2}{4b^6}\right.^{1/2}$$

Thus, the new dissipative term is equivalent to an effective mass that arises due to the collaborative acceleration of the particle This means that the cause for radiation reaction comes directly from the use of the local clock to formulate Maxwell's equations Thus, in this approach, there is no need to assume self-interaction (along with mass renormalization) in order to account for it, as has been required when the observer clock is used

3 6 *Proper-time group*

We now identify the new transformation group that preserves the first postulate of the special theory The standard (Lorentz) time transformations between two inertial observers can be written as

$$t' = \gamma(\mathbf{v})\left[t - \mathbf{x}\cdot\mathbf{v}/c^2\right], \qquad t = \gamma(\mathbf{v})\left[t' + \mathbf{x}'\cdot\mathbf{v}/c^2\right] \tag{11}$$

We want to replace t, t' by τ To do this, use the relationship between dt and $d\tau$ to get

$$t = \frac{1}{c}\int_0^\tau b(s)\,ds = \frac{1}{c}\,\bar{b}\tau, \qquad t' = \frac{1}{c}\int_0^\tau b'(s)\,ds = \frac{1}{c}\,\bar{b}'\tau, \tag{12}$$

where we have used the mean value theorem of calculus to obtain the end result, so that both $\bar{b}$ and $\bar{b}'$ represent an earlier τ-value of b and b' respectively Note that, as b and b' depend on τ, the transformations (12) represent explicit nonlinear relationships between t, t' and τ (during interaction) (This is to be expected in the general case when the system is acted on by external forces) If we set

$$\mathbf{d}^* = \mathbf{d}/\gamma(\mathbf{v}) - (1-\gamma(\mathbf{v}))\left[(\mathbf{v}\cdot\mathbf{d})/\left(\gamma(\mathbf{v})\mathbf{v}^2\right)\right]\mathbf{v},$$

we can write the transformations that fix τ as

$$\mathbf{x}' = \gamma(\mathbf{v})\left[\mathbf{x}^* - (\mathbf{v}/c)\,\bar{b}\tau\right], \qquad \mathbf{x} = \gamma(\mathbf{v})\left[\mathbf{x}'^* + (\mathbf{v}/c)\,\bar{b}'\tau\right],$$

$$\mathbf{u}' = \gamma(\mathbf{v})\left[\mathbf{u}^* - (\mathbf{v}/c)\,b\right], \qquad \mathbf{u} = \gamma(\mathbf{v})\left[\mathbf{u}'^* + (\mathbf{v}/c)\,b'\right], \tag{13}$$

$$\mathbf{a}' = \gamma(v)\left\{\mathbf{a}^* - \mathbf{v}\left[(\mathbf{u}\cdot\mathbf{a})/(bc)\right]\right\}, \qquad \mathbf{a}' = \gamma(\mathbf{v})\left\{\mathbf{a}'^* + \mathbf{v}\left[(\mathbf{u}'\cdot\mathbf{a}')/(b'c)\right]\right\}$$

If we put eq (12) in (11), differentiate with respect to τ and cancel the extra factor of c, we get the transformations between b and b'

$$b'(\tau) = \gamma(\mathbf{v})\left[b(\tau) - \mathbf{u}\cdot\mathbf{v}/c\right], \qquad b(\tau) = \gamma(\mathbf{v})\left[b'(\tau) - \mathbf{u}'\cdot\mathbf{v}/c\right] \tag{14}$$

Eqs (13) in (14) provide an explicit nonlinear representation of the Lorentz group, which uses the local clock to describe the dynamics of the system and preserves the first postulate of the special theory (the only one that really matters)

3 7 Proper-time covariance of Maxwell's equations

In this section, we show explicitly that Maxwell's equations are covariant under our proper-time group For convenience, write the field equations in four-dimensional form using

$$F = \begin{bmatrix} 0 & B_z & -B_y & -iE_x \\ -B_z & 0 & B_x & -iE_y \\ B_y & -B_x & 0 & -iE_z \\ iE_x & iE_y & iE_z & 0 \end{bmatrix}, \quad \frac{\partial}{\partial x_4} = \frac{-i}{b}\frac{\partial}{\partial \tau} \tag{15}$$

It follows that

$$\frac{\partial F_{\alpha\beta}}{\partial x_\gamma} + \frac{\partial F_{\beta\gamma}}{\partial x_\alpha} + \frac{\partial F_{\gamma\alpha}}{\partial x_\beta} = 0, \quad (\alpha, \beta, \gamma = 1, 2, 3, 4), \tag{16}$$

is equivalent to the sourceless equations and

$$\frac{\partial F_{\alpha\beta}}{\partial x_\beta} = \frac{4\pi}{b} J_\alpha, \quad J_\alpha = (J_x, J_y, J_z, ib\rho), \tag{17}$$

is equivalent to the proper-time equations with sources It should be noted that, in (16) and (17) and in the sequel, the summation convention is in force for repeated indices If we define our coefficient matrix $[a_{\mu\nu}]$ by

$$[a_{\mu\nu}] = \begin{bmatrix} 1+(\gamma-1)(v_x^2/v^2) & (\gamma-1)(v_x v_y/v^2) & (\gamma-1)(v_x v_z/v^2) & i\gamma \frac{v_x}{c} \\ (\gamma-1)(v_x v_y/v^2) & 1+(\gamma-1)(v_y^2/v^2) & (\gamma-1)(v_y v_z/v^2) & i\gamma \frac{v_y}{c} \\ (\gamma-1)(v_x v_z/v^2) & (\gamma-1)(v_y v_z/v^2) & 1+(\gamma-1)(v_z^2/v^2) & i\gamma \frac{v_z}{c} \\ -i\gamma \frac{v_x}{c} & -i\gamma \frac{v_y}{c} & -i\gamma \frac{v_z}{c} & \gamma \end{bmatrix},$$

with $\gamma = \left[1-(v/c)^2\right]^{-1/2}$, then the transformations

$$x'_{\mu} = a_{\mu\nu} x_{\nu} \;\left(\mu, \nu = 1, 2, 3, 4\right), \tag{18}$$

correspond for μ = 1, 2, 3 to the first set of equations in (13) with $x_4 = i\bar{b}_{\tau}\tau = i\int_0^{\tau} b(s)ds$ Integrating the first equation in (14), we have

$$\int_0^{\tau} b'(s)ds = \gamma(\mathbf{v})\left[\int_0^{\tau} b(s)ds - \frac{\mathbf{x} \cdot \mathbf{v}}{c}\right] \tag{19}$$

Since the transformations are equivalent to our proper-time transformations, we can transform the fields between observers using the four-vector approach just as is commonly done using Lorentz transformations Thus, we see that the transformations $F'_{\mu\nu} = a_{\mu\alpha} a_{\nu\beta} F_{\alpha\beta}$ $\left(\mu, \nu, \alpha, \beta = 1, 2, 3, 4\right)$ are equivalent to

$$\mathbf{E}' = \gamma\left[\mathbf{E} + \frac{1}{c}\left(\mathbf{v} \times \mathbf{B}\right)\right] - \left(\gamma - 1\right)\frac{\left(\mathbf{E} \cdot \mathbf{v}\right)}{v^2}\mathbf{v}, \tag{20}$$

$$\mathbf{B}' = \gamma\left[\mathbf{B} - \frac{1}{c}\left(\mathbf{v} \times \mathbf{E}\right)\right] - \left(\gamma - 1\right)\frac{\left(\mathbf{B} \cdot \mathbf{v}\right)}{v^2}\mathbf{v} \tag{21}$$

It should not be surprising that eqs (20) and (21) are the same as would be obtained if our observers used their own clocks This is because the transformation coefficient matrix is the same as is used for standard Lorentz transformations between fields On the other hand, when we look at the current and charge densities, the transformations $J'_{\mu} = a_{\mu\alpha} J_{\alpha}$ $(\mu, \alpha = 1, 2, 3, 4)$ are equivalent to

$$\mathbf{J}' = \mathbf{J} + \left(\gamma - 1\right)\frac{\left(\mathbf{J} \cdot \mathbf{v}\right)}{v^2}\mathbf{v} - \gamma\frac{b}{c}\rho\mathbf{v}, \tag{22}$$

$$b'\rho' = \gamma(\mathbf{v})\left[b\rho - \left(\mathbf{J} \cdot \mathbf{v}/c\right)\right] \tag{23}$$

Using the first equation of (12) in (23), we get

$$\rho' = \frac{\rho - \left(\mathbf{J} \cdot \mathbf{v}/bc\right)}{1 - \left(\mathbf{u} \cdot \mathbf{v}/bc\right)} \tag{24}$$

This result is different from the standard one, (which we obtain if we set $b' = b = c$ in (23)),

$$\rho' = \gamma(\mathbf{v})\left[\rho - \left(\mathbf{J} \cdot \mathbf{v}/c^2\right)\right]$$

To see a further difference, if we insert the expression $J/c = \rho(u/b)$ for the current density in (24), we obtain

$$\rho = \rho \frac{1-\left(\boldsymbol{u}\ \boldsymbol{v}/b^2\right)}{1-\left(\boldsymbol{u}\ \boldsymbol{v}/bc\right)} \tag{25}$$

We obtain the following interesting result from eq (25)

Theorem 1 If a source is at rest in the X *frame, then* $\rho = \rho'$ *for all other observers*

Proof The proof is easy, just note that, if $\boldsymbol{u}$ = 0 in X, then b = c and, from equation (25), $\rho = \rho'$ Since X^1 is arbitrary, the result is true for all observers

The above theorem means that, in the proper-time formulation, a spherical charge distribution at rest in any inertial frame will appear spherical to all other inertial observers

It follows that Maxwell's equations are explicitly left covariant under the group transformations induced by eqs (13) and (14) *We now see clearly that the velocity of electromagnetic fields depends on the motion of the source, has a magnitude that is always greater than or equal to c and is not the same for all observers* This may seem strange and even contradictory relative to the second postulate, but it is not The second postulate explicitly assumes that the observer's clock is the natural one to use in measuring the time associated with the observed system Thus, there is no contradiction, but a reflection of the two possible conventions available for the choice of clocks

4. Speed of light problems

In the following sections, we take a brief look at two areas where the use of the observer's clock has run into problems In preparation for this, we pause to give additional consideration to the physical implications of our interpretation of b and b' as the speed of light relative to the source for the different observers (collaborative speed of light) In order to gain some perspective, suppose an emitting system is at rest in the unprimed frame so that b = c In this case, the collaborative speed of light observed in the primed frame is $b' = \gamma(\boldsymbol{v})\,\mathbf{c}$ (vector) and the velocity of the source is seen as $\boldsymbol{u}' = -\gamma(\boldsymbol{v})\boldsymbol{v}$ Thus, if the two observers are separating at high speeds both b' and $\boldsymbol{u}'$ may be very large On the other hand, since the system is at rest in an inertial frame, its proper clock is the same as that of the unprimed observer Thus, the primed observer can obtain the true velocity by using the proper-time group

There are some experiments where use of the observer's clock provides a clear answer A classic example is the Michelson-Morley experiment This experiment gave the first bell of doom for the ether theory, and is easily explained by the special theory (using the clock of the observer) It also has a simple explanation when the clock of the source is used since, in this case, the source is at rest in the frame of the observer so that $\boldsymbol{u} = \mathbf{0} \Rightarrow b = c$

It is clear that, at the speeds obtained in the world of our ordinary experience, no significant difference between the two approaches is expected. However, at high energies, we expect differences to show up in a dramatic way. Indeed they have, but our definition of velocity depends on the clock attached to the observer, $\mathbf{w} = dx/dt$, and all contrary results are interpreted as due to time dilation. Indeed, without this switch in clocks, there is no way to explain the results.

An equally valid interpretation is that the velocity of the system is not $\mathbf{w}$, but $\mathbf{u} = dx/d\tau$ and, in this case, no contrary results occur. The use of $\mathbf{w}$ is clearly a convenient choice for most of ordinary physics (where both choices are the same). However, in high-energy experiments, the local clock of the system is necessary (and used) to determine both when and where to set up particle detectors to record scattering events. The data is then analyzed using time dilation to make the results correspond to velocities below c.

In order to obtain a different view of experiments on the lifetime of fast mesons and the velocity of γ rays and light from moving sources, first consider the definition of momentum. When the clock of the observer is used to measure time, momentum increase is attributed to relativistic mass increase so that

$$\mathbf{p} = m\mathbf{w}, \quad m = m_0\left[1 - w^2/c^2\right]^{-1/2}.$$

On the other hand, if we use the clock of the source, we have that

$$\mathbf{p} = m_0\mathbf{u}, \quad \mathbf{u} = \mathbf{w}\left[1 - w^2/c^2\right]^{-1/2},$$

so that, in this case there is no mass increase, the (proper) velocity increases. It now follows that, in particle experiments, the particle has a fixed mass and invariant decay constant, independent of its velocity, but can have speeds that are much larger than c. An analysis of experiments on the lifetime of fast mesons and the velocity of γ rays and light from moving sources reveal that, at some point, either the speed of light is assumed to be independent of the motion of the source, or time dilation is used. Both of these concepts imply that the clock of the observer is used to measure time. *Thus, these experiments validate the conventional theory but do not prove that the speed of light is c.*

4.1. Relativistic jets in our galaxy :

In 1918 Curtis [23] made the first discovery of jet-like features emanating from the nuclei of galaxies. He identified a jet in the optical range from an elliptical galaxy in the Virgo cluster (M87). Since then, a large number of objects with a jet-like structure have been discovered. However, starting about 30 years ago, researchers began to find quasars with jet expansions of up to ten or more times the speed of light (see Pearson and Zensus [24]; Zensus [25]; Mirabel and Rodriguez [26]). This started quite a stir and led many to suggest the possible breakdown of the special theory. Since the source appeared to be at rest relative to earth, unlike the decay of fast muons from the top of the atmosphere, the assumption of time-dilation would not work. However, Rees [27] suggested that we may be looking at the jets from some angle relative to the observer. He showed how these large speeds may be an aberration because we were looking at a projection of the true image onto the plane of view of the observer. This would explain the apparent approaching and receding condensations with very different velocities.

However, this is an additional assumption which has no independent (experimental) verification and has not been completely accepted as the final solution (see Mirabel and Rodríguez [26]; De Rújula [28]). If our formulation is correct, it is also possible that the true speeds are the ones measured, but the speed of light is different for the approaching and receding condensations, which are caused by two different physical mechanisms.

4.2. Ultra-high energy cosmic rays :

The nature and origin of ultra-high energy cosmic rays (UHECR) continues to cause controversy and concern. One year after Penzias and Wilson [29] discovered the cosmic microwave background radiation (CMBR), Greisen [30] and Zatsepin and Kuzmin [3], estimated that the mean free path of an energetic 10^{19} eV proton moving through the CMBR would be less than the size of our galaxy. From this work, it was expected that all protons with energies above about 4×10^{19} eV (GZK cutoff energy) would be suppressed by dissipative losses in the CMBR. (This limits how far protons can travel to about 100 Mpc.) Thus, it was a real surprise when the Akeno Giant Air Shower Array (AGASA) Collaboration [32] reported observations of a large flux of UHECR with energies above 10^{20} eV. The HiRes (fluorescence detector) Collaboration group [33] published results that appear consistent with support for a cut-off. Additional studies are currently being conducted at the Pierre Auger Observatory designed to resolve the discrepancy. If the AGASA findings are confirmed, new physics may almost surely be required to explain them. (For a recent review see Sigl [34]).

With the conventional definition of velocity, it is very difficult to imagine even the most powerful astrophysical systems, such as active galactic nuclei and/or radio galaxies, accelerating heavy nuclei or protons to the required high energies within existing physical theories. However, if the local clock of the ejecting system determines the distance travelled and speed of a particle, then the GZK cut-off will not hold. Once again, many parts of experimental physics will not be affected since most measurements (physical and astronomical) are based on the dimensionless ratio $\beta = w/c \equiv u/b$. (This also means that light may reach us from much farther away and with more intensity than is traditionally expected. Thus, we may be looking at some galaxies that are not as close and others that are not as far as predicted from conventional theory.)

Other Research. We have also made inroads on some of the basic physical and mathematical problems in relativistic quantum theory.

Dirac Equation. In [35], we have constructed an analytical separation (diagonalization) of the full (minimal coupling) Dirac equation into particle and antiparticle components. The diagonalization is analytic in that it is achieved without transforming the wave functions, as is done by the Foldy-Wouthuysen method, and reveals the nonlocal time behaviour of the particle-antiparticle relationship We then showed explicitly that the Pauli equation is not completely valid for the study of the Dirac hydrogen atom problem in *s*-states (hyperfine splitting), which is used for input to QED We concluded that there are some open mathematical problems with any attempt to explicitly show that the Dirac equation is insufficient to explain the full hydrogen spectrum

Using a new method, we were able to effect separation of variables for full coupling (Coulomb and magnetic dipole), solve the radial eigenvalue problem and provide graphs of the probability density function for the 2*p* and 2*s*-states, and compare them with those of the Dirac-Coulomb case However, at that time, we were not able to solve the angular eigenvalue problem This would have allowed us to provide exact values for the hyperfine splitting separation *via* the Dirac theory, which would provide precise input to QED calculations

Square-Root equation In [36], we have used the theory of fractional powers of linear operators (developed by researchers in probability theory) to construct a general (analytic) representation theory for the square-root energy operator of relativistic quantum theory which is valid for all values of the spin Recall that, it was the inability to understand this operator that lead to the Dirac equation Our general representation is uniquely determined by the Green's function for the corresponding Schrödinger equation We find that, in general, the operator has a representation as a (spacial) nonlocal composite of, at least, three singularities (divergent integrals) In the standard interpretation, the particle component has two negative parts and one (hard core) negative part This effect is confined within a Compton wavelength such that, at the point of singularity, they cancel each other providing a finite result Furthermore, the operator looks like the identity outside a few Compton wavelengths (cutoff) To our knowledge, this is the first example of a physically relevant operator with these properties (Recall that the square-root operator is related to the Dirac operator by a unitary transformation, but one is a spatially nonlocal, while the other is time nonlocal Thus, mathematical equivalence is not physical equivalence)

We also derived an alternate relationship between the Dirac equation (with minimal coupling) and the square-root equation which is much closer than the one obtained *via* the Foldy-Wouthuysen method in that there is no change in the wave function This approach leads to a new Klein-Gordon equation and a new squareroot equation, both of which have the same eigenfunctions and (related) eigenvalues as the Dirac equation Finally, we developed a new perturbation theory that will allow us to extend the range of our theory to include suitable spacetime-dependent potentials

Mathematical problems In [37], we developed a constructive theory of the Feynman operator calculus, which was motivated by, and has direct applications to physics We then developed a general perturbation theory and used it to prove that all theories generated by unitary groups are asymptotic in the operator-valued sense of Poincaré. This proves Dyson's open conjecture about the renormalized series in QED. We showed that our theory can be reformulated as a physically motivated sum over paths, and used this version to prove Dyson's second open conjecture, that the divergences of QED are caused by a violation of the time-energy uncertainty relations.

In [38], we introduce a new Hilbert space which allows us to construct the elementary path integral in the manner originally envisioned by Feynman. We suggest that this Hilbert space is a more appropriate for quantum theory, in that it satisfies the requirements for the Feynman, Heisenberg and Schrödinger representations, while the conventional choice only satisfies the requirements for the Heisenberg and Schrödinger representations.

5. Conclusions

In this paper, we have provided an outline of the research program at Howard University on our approach to a physically motivated representation of classical electrodynamics, along with a brief review of our work on relativistic quantum theory and physically relevant mathematical problems.

The following conclusions may be drawn from this review :

1. The special theory of relativity has another representation in which, the invariant speed of light c is replaced by the invariant local clock of the observed system In this formulation, the new (collaborative) speed of light is not invariant but depends on the motion of the observed system.

2. Maxwell's equations are not unique but have a mathematically equivalent formulation, which is not physically equivalent. Thus, mathematical equivalence is not necessarily physical equivalence. This lack of a direct relationship with the mathematical *versus* the requirements for physical equivalence is also revealed in the time-nonlocal nature of the Dirac operator *versus* the spatially nonlocal nature of the square-root operator. (However, they are related *via* a unitary transformation.)

3. It is possible to have a formulation of classical electrodynamics, that does not depend on the structure of charged particles and does not require self-energy, advanced potentials, mass renormalization, or the problematic Lorentz-Dirac equation, in order to account for radiation reaction.

4. It is possible to have a formulation of the special theory in which the mathematically elegant but counter-intuitive Minkowski four geometry is replaced by the normal Euclidean geometry of our (physical) experience.

5. All experiments and observations based on the assumed constant speed of light c need reevaluation. For example, how far cosmic rays can travel, how far we are from the distant galaxies and the age of the universe.

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